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KALUZA-KLEIN BLACK HOLES WITHIN HETEROTIC STRING THEORY ON A TORUS

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Abstract

We point out that in heterotic string theory compactified on a 6-torus, after a consistent truncation of the 10-d gauge fields and the antisymmetric tensor fields, 4-dimensional black holes of Kaluza-Klein theory on a 6-torus constitute a subset of solutions.

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In this note we show that 4-dimensional (4-d) black holes (BH) of Kaluza-Klein (KK) theory constitute a subset [1] - [5] of 4-d BH solutions of an effective heterotic string theory compactified on a 6-torus [6].

An effective 4-d action for the massless bosonic sector of heterotic string vacua compactified on a 6-d torus is obtained [7] by compactifying the (massless) bosonic part of $D=10$ $N=1$ supergravity coupled to $N=1$ super Yang-Mills theory, containing the dilaton $\Phi^{(10)}$, the 2-form field $B^{(10)}_{\hat{\mu}\hat{\nu}}$ and 16 Abelian gauge fields $A^{(10)\,I}_{\hat{\mu}}$ ($I=1,...,16$), on a 6-torus [1]:

$$S=\int \mathrm{d}^4x\sqrt{-G}\mathrm{e}^{-\Phi}[\mathcal{R}_G+\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}-F^a_{\mu\nu}(LML)_{ab}(F^b)^{\mu\nu}+\frac{1}{8}\mathrm{Tr}(\partial_\mu ML\partial^\mu ML)]\ , \tag{1}$$

where $F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu$ and $H_{\mu\nu\rho}=(\partial_\mu B_{\nu\rho}+2A^a_\mu L_{ab}F^b_{\nu\rho})+permutations$ ($a,b=1,...,28$). $G\equiv \mathrm{det}G_{\mu\nu}$ and the Ricci scalar $\mathcal{R}_G$ are defined in terms of the string metric $G_{\mu\nu}$. M is the $O(6,22)$ matrix of the following 28 scalar fields: the internal part of the 10-d metric $\hat{G}_{mn}\equiv G^{(10)}_{m+3,n+3}$ ($m,n=1,...,6$), "antisymmetric" background fields $B_{mn}\equiv B^{(10)}_{m+3,n+3}$ ($m,n=1,...,6$) and "gauge" background fields $A^I_m\equiv A^{(10)\,I}_{m+3}$ ($m=1,...,6,\quad I=1,...,16$). M has the properties:

$$MLM^T=L,\qquad M^T=M,\qquad L=\begin{pmatrix}0 & I_6 & 0\\ I_6 & 0 & 0\\ 0 & 0 & -I_{16}\end{pmatrix}\ , \tag{2}$$

where I_n is the $n\times n$ identity matrix. L is the matrix invariant under $O(6,22)$ transformations. The 4-d dilaton field $\Phi\equiv\Phi^{(10)}-\frac{1}{2}\ln\det\hat{G}$ is defined in terms of the 10-d dilaton field $\Phi^{(10)}$ and determinant of the internal metric $\hat{G}_{mn}$. Gauge fields $A^m_\mu\equiv\frac{1}{2}\hat{G}^{mn}G^{(10)}_{n+3,\mu}$ ($m,n=1,...,6$) are related to the off-diagonal components of the 10-d metric. Gauge fields A^a_μ with $a=7,...,28$ are related to the off-diagonal components of the 10-d anti-symmetric tensor $B^{(10)}_{\hat{\mu}\hat{\nu}}$ and the 4-d space-time components of the 10-d gauge fields $A^I_{\hat{\mu}}$.

We choose to set the 10-d Abelian gauge fields and 10-d 2-form fields equal to zero; this choice is consistent with the equations of motion in the corresponding 10-d supergravity theory, and thus with the equations of motion of the 4-d effective action (1). Consequently, a consistent truncation of (1) corresponds to setting the anti-symmetric tensor field $B_{\mu\nu}$, a set of 4-d gauge fields A^a_μ (a=7,...,28), as well as the scalar background fields B_{mn}, and A^I_m to zero. The action (1) then reduces to the following form:

$$S=\int \mathrm{d}^4x\sqrt{-G}\mathrm{e}^{-\Phi}[\mathcal{R}_G+\partial_\mu\Phi\partial^\mu\Phi-F^a_{\mu\nu}(LML)_{ab}(F^b)^{\mu\nu}+\frac{1}{8}\mathrm{Tr}(\partial_\nu ML\partial^\mu ML)]\ , \tag{3}$$

where

[1]For notational conventions and the relationship of the 4-d massless modes to the bosonic modes of the corresponding 10-d $N=1$ supergravity theory see for example Ref. [8].

$$M = \begin{pmatrix} \hat{G}^{-1} & 0 & 0 \\ 0 & \hat{G} & 0 \\ 0 & 0 & I_{16} \end{pmatrix} . \tag{4}$$

depends only on $\hat{G}_{mn}$, a real symmetric (6×6)-matrix of scalar fields associated with the internal metric of 6-tori. The action (3) can now be written explicitly as:

$$\begin{aligned} S \quad &= \int \mathrm{d}^4x\sqrt{-G}\mathrm{e}^{-\Phi}\left[\mathcal{R}_G + \partial_\mu\Phi\partial^\mu\Phi - \tfrac{1}{4}\hat{G}_{mn}F^m_{\mu\nu}F^{n\mu\nu} + \tfrac{1}{4}\partial_\mu\hat{G}_{mn}\partial^\mu\hat{G}^{mn}\right] \\ &= \int \mathrm{d}^4x\sqrt{-g}\left[\mathcal{R}_g - \tfrac{1}{2}\partial_\mu\tilde{\varphi}\partial^\mu\tilde{\varphi} - \tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \tfrac{1}{4}\mathrm{e}^{\alpha\varphi}\rho_{mn}F^m_{\mu\nu}F^{n\mu\nu} + \tfrac{1}{4}\partial_\mu\rho_{mn}\partial^\mu\rho^{mn}\right] , \end{aligned} \tag{5}$$

where ρ_{mn} is the unimodular part of the metric $\hat{G}_{mn}$, $\varphi = \frac{1}{\alpha}(\frac{1}{n}\ln\det\hat{G} - \Phi)$, and $\tilde{\varphi} \equiv \frac{1}{\alpha}(\sqrt{\frac{1}{2n}}\ln\det\hat{G} + \sqrt{\frac{2}{n}}\Phi)$. Here, $\rho^{mn}\rho_{n\ell} = \delta^m_\ell$ and $\alpha = \sqrt{\frac{n+2}{n}}$ with $n = 6$. The scalar curvature $\mathcal{R}_g$ and $g = \det g_{\mu\nu}$ are expressed in terms of the Einstein-frame metric $g_{\mu\nu}$.

The action (5) is that of 11-d KK theory compactified on a 7-torus, where the gauge field A^7_μ associated with the seventh torus is turned off. Consequently, the field $\tilde{\varphi} = \sqrt{\frac{2}{n+2}}\Phi^{(10)}$, parameterizing the size of the seventh torus, decouples (except for 4-d gravity) from the other fields and can therefore be set to a constant. This result is obvious, once one realizes that the bosonic sector of 10-d $N = 1$ supergravity with the 10-d gauge fields and antisymmetric tensor field turned off corresponds to 11-d KK theory compactified down to 10-d with the 10-d gauge field, associated with the compactified dimension, turned off.

Thus, the action (5) is effectively that of 10-d KK theory compactified on a 6-torus [9]. The corresponding 4-d BH solutions of (5) are then those of $(4+n)$-d (n=6) KK theory. In particular, with further consistent truncations of the gauge fields, *i.e.*, $A^m_\mu = 0$ ($m = 1, ..., k(< 6)$), (5) reduces to the effective action of $(10-k)$-d KK theory. Namely, the corresponding internal metric fields, *i.e.*, combinations of φ and ρ_{mn} (m or $n \in \{1, ..., k\}$), decouple from the other fields (except for 4-d gravity) and can thus be set to constant values. Specifically, for the choice of $k = 5$ (only one non-zero gauge field) (5) reduces to the action of an effective 5-d KK theory with the corresponding 4-d KK BH solutions [1], as discovered by Duff *et al.* [10].

The supersymmetric embedding of the bosonic action (5) allows one to derive the Bogomol'nyi bound for the ADM mass of the above class of spherically symmetric BH solutions. Among them the supersymmetric ones, *i.e.*, those which preserve (constrained) supersymmetry, can be regarded as nontrivial vacuum configurations, since they saturate the corresponding Bogomol'nyi bounds. Supersymmetric embedding [2] of 4-d Abelian KK BH's with diagonal internal metric Ansatz has been carried out [3] within $(4+n)$-d KK theory ($1 \le n \le 11$), and can thus be applied to BH solutions of (5) as well. Such supersymmetric BH's have at most one magnetic (P) and one electric (Q) charge arising from different $U(1)$'s, thus corresponding to solutions in the effective 6-d KK theory with the internal isometry $U(1)_M \times U(1)_E$. Embedding of (5) in 10-d $N = 1$ supergravity ensures [3]

[2]The embedding is a generalization of a supersymmetric embedding for the BH solutions in 5-d KK theory, found by Gibbons and Perry [11].

that the resulting vacuum configuration preserves one ($N = 1$) of $N = 4$ supersymmetries of the effective 4-d action.

The corresponding non-extreme solutions with a diagonal internal metric [4,5] as well as a class of those with a non-diagonal internal metric [5] have also been found. The latter ones can be obtained [5] as solutions of (5), by performing $SO(n)$ ($n = 6$) rotations on the solutions with a diagonal internal metric. This $SO(6)$ symmetry is realized as a subset of $O(6, 22)$ symmetry [12] of (1).

The class of solutions generated by the $SO(6)$ transformations on the $U(1)_M \times U(1)_E$ BH solutions corresponds [5] to charged configurations $\{P^i, Q^i\}$ ($i = 1, ..., n(= 6)$) subject to the constraint $\sum_{i=1}^{n} P^i Q^i = 0$. The most general solutions within this class, *i.e.*, those with unconstrained charge configurations, are expected to be generated [13] by (one parameter) transformations, belonging to $SO(2, n)$ ($n = 6$), on the former solutions. Here, $SO(2, n)$ is a symmetry of the effective 3-d Lagrangian density [2] for the spherically symmetric BH Ansatz in $(4 + n)$-d KK theory.

Such explicit solutions for all static 4-d KK BH's would in turn allow for a study of their global space-time and thermal properties. They would in turn provide a sub-class of solutions for general 57 [or 58]-parameter dyonic [or rotating] BH solutions which could be generated [6] by $[O(22, 2) \times O(6, 2)]/[O(22) \times O(6) \times SO(2)]$ transformations on the 4-d Schwarzschild [or Kerr] solution.

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